# Satisficing under Limited Attention*

Dayang Li †

Feb 11, 2023

**Abstract**

Satisficing choice pattern has been accused of a lack of cognitive ability in perceiving and analyzing alternatives. In an effort to disentangle the limitations of noticing alternatives from other factors, this paper proposes a model of satisficing under limited attention. Our focus centers on the idempotent attention rules, leading to Satisficing under Idempotent Attention (SIA). Our study provides a characterization of SIA as well as a discussion of revealed attention and preference practices. Notably, these outcomes stem from choices made on a subset of menus. The revealed attention and preference, however, remain inherently non-unique. Additionally, considering the idempotent nature of attention filters and competition filters, we also present distinctive characterizations of satisficing under these two attention rules.

**JEL Codes:** D01; D91

**Keywords:** Satisficing; Bounded rationality; Limited attention; Idempotent attention

*I am hugely indebted to Hiroki Nishimura for his support and guidance. I am grateful to Siyang Xiong and Urmee Khan for their suggestions and encouragement. I thank the seminar participants at UC Riverside, AMES in China 2023, and the Xiangjiang River Forum in Economics, Finance and Management for Young Scholars 2023. Of course, all errors are my own.

†University of California Riverside, Department of Economics, 900 University Avenue, Riverside, CA 92551. Email: dayang.li@email.ucr.edu.

# 1 Introduction

The classical theory of choice proposes that decision makers (DMs) choose the "best" alternative from a given set of choices. In real life scenarios, however, the best alternative may not always be the one chosen. Simon (1959) argues: " The entrepreneur may not care to maximize, but may simply want to earn a return that he regards as satisfactory."

Satisficing, as described in Simon (1997), refers to the selection of alternatives that meet or exceed the criteria instead of selecting the best alternative. Simon (1997) ascribe satisficing to "the limits of human cognitive capacity for discovering alternatives, computing their consequences under certainty or uncertainty, and making comparisons among them."

The lack of cognitive abilities has been used as motivation in previous research related to satisficing. It has been demonstrated in the research on intransitive indifference (for example, see Luce (1956), Fishburn (1975), Aleskerov et al. (2007), and Manzini and Mariotti (2012)) that DMs are incapable of noticing small differences in utility between two alternatives. As outlined in Tyson (2008), DMs are unable to recognize some preferences among alternative options due to their limitations in cognitive ability. According to Frick (2016), DMs have limited cognitive resources, causing them to perceive coarse preferences in a large menu.

The reasons for this can be divided into two categories: the ability to perceive alternatives and the ability to analyze them. Simon (1982) uses an example of chess players to illustrate it. Chess players may notice a limited number of moves at any given time. Analyzing them sequentially, they select a move once it meets their objectives. The process of analyzing moves involves the complexity of computing, which is difficult to model. DMs' limited ability to perceive alternatives, however, can be modeled separately from the classical satisfying model.

This limited ability of DMs to recognize alternatives is referred to as limited attention. Assuming limited attention, satisficing becomes that a DM pays attention to a nonempty subset of the choice set, and selects options that meet or exceed certain criteria.

Satisficing under limited attention (SLA) is comprised of three components: attention, preferences, and criteria. Let $\Gamma$ be the attention rule of the DM. It maps every choice set to one of its nonempty subsets. The DM pays attention to $\Gamma(S)$ given a choice set $S$. The preference is a binary relation $\succcurlyeq$ on alternatives which is assumed to be a linear order, i.e., complete, transitive and antisymmetric. The DM has a criterion for every choice set $S$, denoted by $\theta(S) \in \Gamma(S)$. We refer to $\theta$ as a threshold function. If the DM's behavior is an

SLA, the chosen alternatives should catch the attention and meet or exceed the criterion. Mathematically, for $S$, the chosen alternatives should in $\{x \in \Gamma(S) : s \succcurlyeq \theta(S)\}$.

Suppose that the set of alternatives is finite. Imagine that we can observe the DM's choice for every choice set as an outsider. The DM's choice in $S$ is denoted by $C(S)$, which is a nonempty subset of $S$. Since the DM can select multiple alternatives, $C$ is a choice correspondence. It follows that a choice correspondence can be rationalized by an SIA if and only if there exists a tuple $\langle \Gamma, \succcurlyeq, \theta \rangle$ such that $C(S) = \{x \in \Gamma(S) : s \succcurlyeq \theta(S)\}$ for every choice set $S$.

In the absence of any further restrictions, every choice correspondence can be rationalized by an SLA. Similar situations can be found in Masatlioglu et al. (2012), Lleras et al. (2017) and Lleras et al. (2021). The papers impose a number of restrictions on attendance rules [1]. We also investigate SLA with respect to a special attention rule, idempotent attention rules (see Li (2023)). According to Li (2023), an attention rule $\Gamma$ is idempotent if $\Gamma(S) = \Gamma(\Gamma(S))$ for every choice set $S$. This implies that the DM believes that $S$ and $\Gamma(S)$ are the same choice set. As the logic of choices within identical sets should be the same, we should ensure that the thresholds of $S$ and $\Gamma(S)$ are the same, i.e., $\theta(S) = \theta(\Gamma(S))$. This model is referred to as satisficing under idempotent attention (SIA).

We provide two methods of characterizing the SIA: the acyclicity of a binary relation and the WARP-like axiom. The former characterization drives from an equivalent statement of satisficing: for a choice set, the picked alternatives should be better than the unchosen ones. It turns out that the acyclicity of this "revealed" binary relation characterizes the satisficing. As well, in the SIA, the chosen options should be better than the alternatives that attract attention but are not chosen.

The WARP-like axiom for satisficing suggests that every menu contains the "best" alternative, and if a non-best alternative is chosen when the best one is available, then the best alternative must be chosen. As an analogy, the WARP-like axion for the SIA requires that the best alternative be selected if the DM pays attention to it.

For characterizing the SIA, the remaining question is how to determine the DM's attention rule. According to the idempotent attention rule, the choice in $S$ is the same as the choice in $\Gamma(S)$. In other words, if some nonempty set of $S$ serves as the $\Gamma(S)$, they must exhibit the same choices. When a set does not have any proper subsets exhibiting the same choice, the DM should pay full attention to it. We borrow the concept from Li

[1] These new attention rules are known as attention filters (see Masatlioglu et al. (2012)), competition filters (see Lleras et al. (2017)), and path independent consideration (see Lleras et al. (2021)), respectively.

(2023), and refer to these sets as basic sets. Consequently, every set $S$ has a corresponding basic set $B$, which can be roughly interpreted as $\Gamma(S)$, i.e., $\Gamma(S) = B$.

Inspired by Bernheim and Rangel (2007) and Bernheim and Rangel (2009), we study the revealed SIA pairs, i.e., $\langle \Gamma, \succcurlyeq, \theta \rangle$ with the consideration of welfare implications. Similar to Masatlioglu et al. (2012), we use the most stringent criterion for revealing SIA pairs to provide the safest option for welfare policies.

It is critical to emphasize that every choice correspondence that can be rationalized by an SIA must have at least two consistent SIA pairs. This is primarily due to the non-unique nature of revealed preferences. It also results in the inability to identify the DM's choice procedure under the SIA in a unique manner.

Attention filters and competition filters are special cases of idempotent attention rules (see Li (2023)). The satisficing under attention filter (SAF) and satisficing under competition filters (SCF) are therefore special cases of the SIA. In this paper, we demonstrate that the SAF, as well as the SCF, can both be characterized by two distinct systems of axioms. These axioms are based on the same intuitions as those in the SIA. The distinctions between these three models are driven by attention formation procedures.

The reaming paper is arranged as follows. The SIA will be formally introduced in Section 2. In Section 3, we provide two characterizations of the SIA as well as the results of revealed SIA pairs. In Section 4, we characterize two special cases of the SIA: SAF and SCF. Section 5 concludes this paper.

## 2 The Model

Let $X$ be a finite set, and $\mathcal{X}$ be the collection of nonempty subsets of $X$. We refer to each element in $X$ as an alternative and each element in $\mathcal{X}$ as a menu. A mapping $C : \mathcal{X} \to \mathcal{X}$ is a choice correspondence if $C(S) \subseteq S$ for all $S \in \mathcal{X}$. An outside researcher can observe the choice made by a decision maker (DM) for each menu, i.e., $C$.

In the satisficing model, DMs have a specific criterion in mind. Given any menu $S$, they only choose the alternatives that are "better" than the criterion. We generalize the idea of criteria as a mapping $\theta : \mathcal{X} \to X$. Further, we say $\theta$ is a threshold function if $\theta(S) \in S$ for all $S \in \mathcal{X}$.

When DMs are limitedly attentive, they pay attention to some (not necessarily all) alternatives in each menu. When they use the same procedure in the satisficing model to make decisions, they should use it with the alternatives that capture their attention.

We refer to this process as satisficing under limited attention. To describe that DMs has limited attention, we define a mapping $\Gamma : \mathcal{X} \to \mathcal{X}$. We say $\Gamma$ is an attention rule if $\Gamma(S) \subseteq S$ for all $S \in \mathcal{X}$.

**Definition 1.** *A choice correspondence $C$ is a **satisficing under limited attention (SLA)** if there is a tuple $\langle \Gamma, \succcurlyeq, \theta \rangle$ such that $C(S) = \{s \in \Gamma(S) : s \succcurlyeq \theta(S)\}$ where*

*(1) $\Gamma : \mathcal{X} \to \mathcal{X}$ is an attention rule;*

*(2) $\succcurlyeq$ is a linear order, i.e., $\succcurlyeq$ is complete, transitive, and antisymmetric;*

*(3) $\theta : \mathcal{X} \to X$ is a threshold function with $\theta(S) = \theta(\Gamma(S)) \in \Gamma(S)$ for all $S \in \mathcal{X}$.*

In this framework, there is another way to interpret the decision under satisficing: The chosen alternatives should be better than the unchosen ones. Under the assumption of limited attention, it can be interpreted as if the DM pays attention to $x$ and $y$ in a menu $S$, $x \in C(S)$ while $y \notin C(S)$ implies that $x \succ y$.

**Proposition 1.** *$C$ can be rationalized by an SLA if and only if there is a linear preference $\succcurlyeq$ and an idempotent attention rule $\Gamma$ such that for all $S \in \mathcal{X}$*

$$C(S) = \begin{cases} \{s \in \Gamma(S) : s \succ x \text{ } for \text{ } all \text{ } x \in \Gamma(S) \setminus C(S)\} & if \text{ } \Gamma(S) \setminus C(S) \neq \emptyset, \\ \Gamma(S) & if \text{ } \Gamma(S) \setminus C(S) = \emptyset. \end{cases}$$

*Proof.* Suppose $\succcurlyeq$ can be rationalized by an SLA under $\langle \Gamma, \succcurlyeq, \theta \rangle$. Fix any $S$, and take any $x \in C(S)$. When $C(S) = \Gamma(S)$, by the SIA, we have that $\theta(S) = \min(\Gamma(S), \succcurlyeq)$. When $C(S) \neq \Gamma(S)$, we know that $x \succ \theta(S) \succ y$ for all $y \in \Gamma(S) \setminus C(S)$. Take any $s \in \Gamma(S)$ with $s \succ x$ for all $x \in \Gamma(S) \setminus C(S)$. If $s = \theta(S)$, then $s \in C(S)$. If $s \neq \theta(S)$ and $\theta(S) \succ s$, then $s \notin C(S)$. We then know that $s \succcurlyeq \theta(S)$, which implies that $s \in C(S)$.

For the converse direction, suppose that there is a linear preference $\succcurlyeq$ and an idempotent attention rule $\Gamma$ such that $C(S)$ is given as in this Proposition. Let $\theta(S) = min(C(S), \succcurlyeq)$ for all $S \in \mathcal{X}$. It is evident that $s \in \Gamma(S) \cap C(S)$ implies that $s \succcurlyeq \theta(S)$ for all $S$. □

Similar to the situation in Masatlioglu et al. (2012), if attention rules are not further restricted, every choice correspondence is an SLA.

**Proposition 2.** *Every choice correspondence $C$ is an SLA.*

*Proof.* Fix an arbitrary linear order $\succcurlyeq$ on $X$. Let $\Gamma(S) = C(S)$, and $\theta(S) = min(C(S), \succcurlyeq)$. Take any $x \in C(S)$, we know that $\theta(S) = min(C(S), \succcurlyeq) \succcurlyeq x$, which implies that $C(S) \subseteq \{x \in \Gamma(S) : x \succcurlyeq \theta(S)\}$. The converse direction is obvious. □

Among all potential restrictions on attention rules, we focus on the family of idempotent attention rules.

**Definition 2.** *(Idempotent Attention Rule) An attention rule* $\Gamma$ *is an idempotent attention rule if* $\Gamma(S) = \Gamma(\Gamma(S))$ *for all* $S \in \mathcal{X}$.

Li (2023) points out that idempotent attention rules are generalizations of attention filters (cf. Masatlioglu et al. (2012)), competition filters (cf. Lleras et al. (2017)), and path independent consideration from Lleras et al. (2021). It is implicit in idempotent attention rules that DMs are not aware of omitted alternatives on a menu $S$. Additionally, they believe that $S$ and $\Gamma(S)$ are identical.

## 3 Satisficing under Idempotent Attention

The satisficing under idempotent attention can be achieved by imposing idempotence on the attention rule of the SLA.

**Definition 3.** *(SIA) A choice correspondence* $C$ *is a* ***satisficing under idempotent attention (SIA)*** *if* $C(S) = \{s \in \Gamma(S) : s \succcurlyeq \theta(S)\}$ *where*

1. $\Gamma$ *is an idempotent attention rule.*

2. $\succcurlyeq$ *is a linear order, i.e.,* $\succcurlyeq$ *is complete, transitive, and antisymmetric.*

3. $\theta : \mathcal{X} \to X$ *is a threshold function such that* $\theta(S) = \theta(\Gamma(S)) \in \Gamma(S)$ *for each* $S \in \mathcal{X}$.

*We infer* $\langle \Gamma, \succcurlyeq, \theta \rangle$ *as an SIA pair that is consistent with* $C$.

When we assume that DMs' attention rules are idempotent, there are some choice correspondences that cannot be rationalized by an SIA.

**Example 1.** *Let* $X = abcd$*, and the choice correspondence is given as*

| $\|S\| \geq 2$ | *abcd* | *abc* | *abd* | *acd* | *bcd* | *ab* | *ac* | *ad* | *bc* | *bd* | *cd* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $C$ | *abc* | *ab* | *a* | *ad* | *bd* | *a* | *a* | *d* | *b* | *b* | *c* |

*Suppose that $\langle \Gamma, \theta, \succcurlyeq \rangle$ is a consistent SLA. Since $C(abcd) = abc \neq C(abc)$, thus $\Gamma(abcd) = abcd$ which implies that $c \succ d$. Similarly, $\Gamma(acd) = acd$. However, $C(acd) = ad$ implies that $d \succ c$.*

## 3.1 The Characterization of SIA

According to idempotent attention rules, the DM pays attention to $\Gamma(S)$ for a menu $S$, all the alternatives in $\Gamma(S)$ should catch the DM's attention when $\Gamma(S)$ is displayed. Due to this, the DM's attention can be reduced to one of its subsets for each menu. Moreover, since $\theta(S) = \theta(\Gamma(S))$, $C(S) = C(\Gamma(S))$ when $C$ can be rationalized by an SIA. The reduction procedure is not applicable to a set $S$ if it does not have a proper subset in which the DM makes the same choice. We refer to $S$ as a basic set.

**Definition 4.** *(Basic Sets) A set $S \in \mathcal{X}$ is **basic** if there is no $T \subset S$ such that $C(T) = C(S)$.*

The collection of basic sets is determined by the choice correspondence of the DM. The collection of basic sets with respect to $C$ is referred to as $\mathcal{B}_C$. Usually, when investigating the choices of the DM and there is no misrepresentation, we use $\mathcal{B}$ as the collection of basic sets.

Each menu $S$ has a basic set $B$ with $B \subseteq S$ where $C(S) = C(B)$. This $B$ is referred to as a corresponding basic set of $S$, and the collection of corresponding basic sets of $S$ is denoted as $\mathcal{B}(S)$. If we want to interpret a $B \in \mathcal{B}(S)$ as $\Gamma(S)$ as the intuition in defining basic sets, $B$ must catch the DM's full attention due to the requirement of idempotent attention rules.

**Proposition 3.** *If $C$ can be rationalized by an SIA under $\langle \Gamma, \succcurlyeq, \theta \rangle$, then $\Gamma(B) = B$ for all $B \in \mathcal{B}$.*

*Proof.* Suppose $C$ is an SIA. Take any consistent pair $\langle \Gamma, \theta, \succcurlyeq \rangle$, and any $B \in \mathcal{B}$. By contradiction, suppose that $\Gamma(B) \subset B$. Notice that, $\theta(\Gamma(B)) = \theta(B)$ because $\Gamma(B) = \Gamma(\Gamma(B))$. We have

$$\begin{aligned} C(\Gamma(B)) &= \{x \in \Gamma(\Gamma(B)) : x \succcurlyeq \theta(\Gamma(B))\} \\ &= \{x \in \Gamma(B) : x \succcurlyeq \theta(B)\} \\ &= C(B). \end{aligned}$$

Hence, $B$ is not basic. That's a contradiction. $\square$

When we interpret the DMs' choice on basic sets as mentioned in Proposition 1, $x, y \in B$, $x \in C(B)$ and $y \notin C(B)$ indicates that $x$ is strictly preferred to $y$.

**Definition 5.** *For any $x, y \in X$, we say $x\ P\ y$ if there is a basic set $B$ with $x, y \in B$ such that $x \in C(B)$ and $y \notin C(B)$.*

When $C$ can be rationalized by an SIA, $x\ P\ y$ must imply that $x$ is strictly preferred to $y$. Due to this, for every linear preference $\succcurlyeq$ that is consistent with the SIA, $P \subseteq \succcurlyeq$. Assuming that $x\ P\ y$ and $y\ P\ z$, we can infer $x\ P\ z$ despite the absence of $B$ where $x \in C(B)$ and $z \notin C(B)$. In a different perspective, if there is a $B$ where the DM's choice shows that $z\ P\ x$, then $C$ cannot be rationalized by an SIA since $\succcurlyeq$ exhibits a cycle.

Aleskerov et al. (2007) and Tyson (2008) provide a characterization of satisficing. Specifically, the DM should prefer the chosen alternative over the unchosen alternative, and this binary relation should be acyclic. In the same manner, the acyclicity of $P$ ensures a satisficing on $\mathcal{B}$.

The SIA is also characterized by the acyclicity of $P$. Due to the fact that $\mathcal{B}(S) \neq \emptyset$, $S$ can be associated with one of its corresponding basic sets by setting $\Gamma(S) = B$. It is possible to make this construction an idempotent attention rule by arbitrary permutations of $\mathcal{B}$. The linear preference in this SIA may be any completion of $P$. Once the $\succcurlyeq$ has been determined, would the threshold function be $min(C(S), \succcurlyeq)$ for all $S \in \mathcal{X}$.

**Lemma 1.** *$C$ is an SIA if and only if $P$ is acyclic.*

*Proof.* We first suppose that $C$ is an SLA under $\langle \Gamma, \succcurlyeq, \theta \rangle$. Since $\Gamma(B) = B$ for all $B \in \mathcal{B}$. By contradiction, suppose there is a collection $\{x_i\}_{i=1}^k$ where $x_i \in X$ for all $i$ such that $x_1\ P\ x_2\ P, ..., P\ x_k\ P\ x_1$. Hence, we know there is a collection of basic sets $\{B_i\}_{i=1}^k$ such that $x_i \in C(B_i)$ and $x_{i+1} \in B_i \setminus C(B_i)$ for all $i < k$, and $x_k \in C(B_k)$ and $x_1 \in B_k \setminus C(B_k)$. As a result, $x_1 \succcurlyeq x_2, ..., \succcurlyeq x_k \succcurlyeq x_1$.

Now supposes $P$ is acyclic. Let $\{B_i\}_{i=1}^n$ be the collection of basic sets. Let $P_R$ be the transitive closure of $P$, $\overline{P_R}$ be a completion of $P_R$. Let $\Gamma(S) = B_{min\{i:\ B_i \subseteq S\ and\ C(B_i) = C(S)\}}$, and $\theta(S) = min(C(S), \overline{P_R})$. We then claim that $C(S) = \{x \in \Gamma(S) :\ x \succcurlyeq \overline{P_R}\ \theta(S)\}$. Take any $x \in C(S)$, we then know that $x \in B_{min\{i:\ B_i \subseteq S\ and\ C(B_i) = C(S)\}} = \Gamma(S)$, and $x\ \overline{P_R}\ \theta(S)$. For the converse, take any $S$ and $y \in \{x \in \Gamma(S) :\ x\ \overline{P_R}\ \theta(S)\}$. By contradiction, suppose that $y \notin C(S)$, we know that $y \in B_{min\{i:\ B_i \subseteq S\ and\ C(B_i) = C(S)\}} = \Gamma(S)$ and $y \notin C(\Gamma(S))$. That's a contradiction. $\square$

According to classical revealed preference theory, a choice function is rationalizable when it satisfies the Weak Axiom of Revealed Preference (WARP). Masatlioglu et al. (2012) states the WARP as:

> "For any nonempty $S$, there exists $x^* \in S$ such that for any $T$ including $x^*$, if $C(T) \in S$; then $C(T) = x^*$."

The intuition behind WARP is that the DM's choice should be consistent with some maximization behavior. The $x^*$ is the "best" alternative in $S$. As a rule, rational DMs should not choose alternatives that are less desirable than the best alternative. [2] From the standpoint of revealing preferences, the chosen alternative should be the best option from the menu. The DM should, of course, prefer it to other alternatives that have not been chosen.

As part of the satisficing framework, DMs do not necessarily need to engage in maximization behaviors. As opposed to selecting the best option in every menu, they tend to set a criterion $\theta$ and choose the options that meet or exceed that criterion. Despite the differences in intuition between classical choice and satisficing models, they share a similar implication regarding preferences: the chosen alternative should be better than the unchosen alternatives. The satisficing model under full attention can be characterized by the **WARP-S:** For any nonempty $S$, there exists $x^* \in S$ such that for any $T$ including $x^*$, if $C(T) \cap S \neq \emptyset$, then $x^* \in C(T)$. [3]

In WARP-S, if an inferior alternative is chosen when there is a better option, then the better option should also be selected. There is an additional requirement for the DM's attention when limited attention is present. Specifically, if $x$ is better than $y$ and both $x$ and $y$ are available, if $y$ is chosen, the DM must choose $x$ if $x$ catches the attention of the DM.

**Axiom 1.** *(WARP-IA) For every $S \in \mathcal{X}$, there is an $x^* \in S$ such that for any basic set $B$ if $C(B) \cap S \neq \emptyset$ and $x^* \in B$ then $x^* \in C(B)$.*

WARP-IA generalizes the idea of WARP-S to cases of limited attention. When we relax the requirement of basic sets in WARP-IA, WARP-IA and WARP-S are identical. A

[2] This version of WARP is of classical choice theory with linear preferences. The best alternatives in each menu are not necessarily unique if the preference is a weak order. DMs' choices are described as choice correspondences in this case. The corresponding WARP becomes: For any $S, T \in \mathcal{X}$, if $C(S) \cap T \neq \emptyset$; then $C(T) \cap S \subseteq C(S)$. Two different versions of WARP have similar meanings. This paper focuses on the version provided by Masatlioglu et al. (2012), which provides a suitable benchmark for comparison.

[3] Please see Appendix A for details.

DM considers all alternatives in a basic set when $\succcurlyeq$ can be rationalized by an SIA based on Proposition 3. Therefore, when the DM chooses a less desirable alternative in $B$, we can infer that better alternatives should also be selected.

The $x^*$ in $S$ mentioned in WARP-IA could be interpreted as the most preferred alternative in $S$ with respect to the DM's preference. Moreover, since the DM's attention on $S$ is one of the corresponding basic sets of $S$, when using WARP-IA we can focus on the collection of $\mathcal{B}$ rather than $\mathcal{X}$. Consequently, WARP-IA is equivalent to the acyclicity of $P$.

**Lemma 2.** *$P$ is acyclic if and only if $C$ satisfies WARP-IA.*

*Proof.* Suppose that $P$ is acyclic. We know that $C$ can be rationalized by an SIA pair $\langle \Gamma, \theta, \succcurlyeq \rangle$. Take any $S \in \mathcal{X}$, and let $x^* = max(S, \succcurlyeq)$. If $B$ is basic, then $\Gamma(B) = B$. Since $C(B) \cap S \neq \emptyset$, we know that $x^* \succcurlyeq y$ for all $y \in C(B) \cap S$. Therefore $x^* \in C(B)$.

For the converse direction, we prove it by the contrapositive statement. Suppose that $P$ is cyclical. There is a $z_1 \in X$ such that $z_1 \; P_R \; z_1$ for a finite collection of alternatives $\{z_j\}_{j=1}^m$. Let $\{Z_j\}_{j=1}^m$ be the collection of basic sets where

$$
\begin{aligned}
& z_1, z_2 \in Z_1, \;\; z_1 \in C(Z_1), \;\; \textit{and} \; z_2 \notin C(Z_1); \\
& z_2, z_3 \in Z_2, \;\; z_2 \in C(Z_2), \;\; \textit{and} \; z_3 \notin C(Z_3); \\
& \dots \\
& z_1, z_m \in Z_m, \;\; z_m \in C(Z_m), \;\; \textit{and} \; z_1 \notin C(Z_m).
\end{aligned}
$$

Now, consider $Z = \{z_j\}_{j=1}^m$. For $z_1 \in Z$, there is a basic set $Z_m$ with $z_1 \in Z_m$ such that $C(Z_m) \cap S \neq \emptyset$ and $z_1 \notin Z_m$. For $z_j \in Z$ where $j > 1$, there is a basic set $Z_{j-1}$ with $z_j \in Z_{j-1}$ such that $C(Z_{j-1}) \cap S \neq \emptyset$ and $z_j \notin Z_{j-1}$. □

Lemma 1 and Lemma 2 suggest that WARP-IA characterizes the SIA.

**Theorem 1.** *A choice correspondence $C$ is an SIA if and only if $C$ satisfies WARP-IA.*

### 3.2 Revealed SIA Pairs

A choice correspondence $C$ might be rationalized by multiple SIA pairs. The situation arises from the fact that there are multiple candidates for linear preferences and idempotent attention rules.

**Example 2.** *Let $X = xyz$. Consider the choice correspondence $C$ and idempotent attention rules*

| $\|S\| \geq 2$ | $xyz$ | $xy$ | $xz$ | $yz$ |
|---|---|---|---|---|
| $C$ | $x$ | $xy$ | $x$ | $y$ |
| $\Gamma_1$ | $x$ | $xy$ | $x$ | $y$ |
| $\Gamma_2$ | $xz$ | $xy$ | $xz$ | $y$ |
| $\Gamma_3$ | $xz$ | $xy$ | $xz$ | $yz$ |
| $\Gamma_4$ | $xyz$ | $xy$ | $xz$ | $yz$ |

*Consider two preferences on $X$: $x \succ_1 y \succ_1 z$ and $x \succ_2 z \succ_2 y$. Let $\theta_i(S) = min(C(S), \succ_i)$ for $i = 1, 2$. There are multiple SIA pairs to represent $C$ such as $\langle \Gamma_1, \succcurlyeq_1, \theta_1 \rangle$, $\langle \Gamma_2, \succcurlyeq_1, \theta_1 \rangle$, $\langle \Gamma_3, \succcurlyeq_2, \theta_2 \rangle$ $\langle \Gamma_4, \succcurlyeq_2, \theta_2 \rangle$.*

*Obviously, when the preference is fixed, only some of the idempotent attention rules are consistent with the SIA. Given the DM's preference as $\succcurlyeq_1$, all idempotent attention rules are plausible, while when the DM's preference is $\succcurlyeq_2$, the $\Gamma_4$ is not consistent.*

*Similarly, when the idempotent attention rule is determined, some of the preference relations and corresponding threshold functions will no longer be plausible. For example, given $\Gamma_2$, the DM should strictly prefer $x$ to $z$. Meanwhile, if the idempotent attention rule is $\Gamma_4$, she should strictly prefer $x$ the most followed by $y$, and $z$ the least.*

Let $\langle \Gamma_i, \succcurlyeq_i, \theta_i \rangle_{i \in I}$ be the collection of all consistent SIA pairs for the choice correspondence $C$. Some linear orders on $X$ should never coincide with $\succcurlyeq_i$ for some $i \in I$. For instance, in Example 1, *abcd* is a basic set which implies that $c\ P\ d$. Therefore, any linear order $\succcurlyeq$ that includes $d \succ c$ cannot be considered consistent. Meanwhile, in Example 2, when the preference is fixed in an SIA pair, the choice of threshold functions and the idempotent attention rules are limited.

Afterwards, we analyze the consistent preferences, then examine the consistent idempotent attention rules, and finally examine the consistent threshold functions.

### 3.2.1 Revealed Preference under the SIA

Multiple preference relations are consistent with the same $C$. There are even some that are controversial. For instance, in Example 2 $\succcurlyeq_1$ and $\succcurlyeq_2$ are all consistent linear orders, whereas $y \succ_1 z$ and $z \succ_1 y$ are controversial.

The implications of these two contradictory predictions for welfare policy are controversial. Despite the fact that policymakers do not intend to harm the DM, if they falsely believe one preference rather than another, the policy may adversely affect the DM. Inspiring by Bernheim and Rangel (2007) and Bernheim and Rangel (2009), we follow Masatlioglu et al. (2012) and use the most stringent criteria to reveal the DM's attention.

**Definition 6.** *Given any $x, y \in X$, $x$ is* ***revealed to be preferred to*** *$y$ if $x \succcurlyeq_i y$ for all $i \in I$.*

According to the proof Lemma 1, any completion of $P_R$, the transitive closure of $P$, can be a consistent linear order. As a result, $P_R$ captures the revealed preference.

**Theorem 2.** *Suppose $C$ admits an SIA. For any $x, y \in X$, $x$ is revealed to be preferred to $y$ if and only if $x \; P_R \; y$.*

*Proof.* Suppose that $C$ admits an SIA. Take any $x, y \in X$ with $x \; P_R \; y$, by the proof of Lemma 1 we know that $x \succ_i y$ for all $i \in I$. For the converse direction, suppose that $x$ is revealed to be preferred to $y$, i.e., $x \succcurlyeq_i y$ for all $i \in I$. By contradiction, suppose that $\neg x \; P_R \; y$. If $y \; P_R \; x$, then $y \succ_i x$ for all $i \in I$. If $y$ and $x$ are $P_R$ incomparable, then there is a completion of $P_R$ such that $y \; \overline{P_R} \; x$. As a result, there is $i \in I$ such that $y \succ_i x$. □

According to Theorem 2, there is an immediate relationship between consistent linear orders and $P_R$.

**Corollary 3.** *If $C$ is an SIA, then $\bigcap_{i \in I} \succcurlyeq_i = P_R$.*

Every $\succcurlyeq_i$ represents a potential underlying preference used by the DM. It can be any completion of $P_R$. It should be noted, however, that uniqueness of preference is never true.

**Proposition 4.** *Suppose that $|X| \geq 2$. If $C$ is an SIA, then there exists a pair of $x, y \in X$ such that $x, y$ are $P_R$ incomparable. In other words, if $C$ is an SIA, then $|I| \geq 2$.*

*Proof.* Suppose that $C$ is an SIA. For any $S \in \mathcal{X}$, if $|C(S)| = 1$, then we can let $\Gamma(S) = C(S)$. Consequently, there is no information for revealed preference. We can focus on the collection of basic sets where $|C(B)| \geq 2$. By contradiction, suppose that every pair of elements in $X$ is $P_R$ comparable. Take the top 2 elements of $X$ with respect to $P_R$, and denote them as $x$ and $y$. Let's consider all basic sets that contain $x$ and $y$. We then know that $\{x, y\} \subseteq C(B)$ for all these kinds of basic sets. As a result, $x$ and $y$ are $P_R$ incomparable. □

The uniqueness of revealed preferences is impeded by two factors. As a first point, satisficing differs from classical utility maximization. By satisficing, the DM selects alternatives that meet or exceed the criteria. The best alternative should be selected, but not all chosen alternatives are necessarily the most desirable. Consider the case in which the DM always chooses $x$ whenever $x$ is available. The same is true for $y$ as well. In this manner, we are able to prove that $x$ and $y$ are all (at least) better than the rest of the alternatives while we are unable to determine the preference order between $x$ and $y$ by analyzing the revealed preferences.

Second, idempotent attention rules make it more difficult to analyze revealed preferences. With full attention, it is possible to conclude that the selected alternatives should be superior to the non-selected ones. As a result, determining the best alternatives is ambiguous. When the DM is limited in their attention, we may not be able to distinguish between the worst alternatives and the best because the unchosen alternatives may either be dominated by other alternatives or neglected by the DM.

**Example 3.** *Let $X = xyz$, and the choice correspondence $C$ is given as*

| $\|S\| \geq 2$ | $xyz$ | $xy$ | $xz$ | $yz$ |
|---|---|---|---|---|
| $C$ | $xy$ | $xy$ | $x$ | $z$ |
| $\Gamma_1$ | $xy$ | $xy$ | $x$ | $z$ |
| $\Gamma_2$ | $xyz$ | $xy$ | $x$ | $z$ |
| $\theta$ | $y$ | $y$ | $x$ | $z$ |

*Consider two distinct linear orders on $X$: $x \succ_1 y \succ_1 z$ and $z \succ_1 x \succ_1 y$. It is observed that $\langle \Gamma_1, \succcurlyeq_1, \theta \rangle$ and $\langle \Gamma_2, \succcurlyeq_2, \theta \rangle$ are all consistent pairs of SIA. It is difficult, however, to tell whether $z$ is the best or worst, which makes the practice of revealed attention in this paper more convincing.*

### 3.2.2 Revealed Attention under SIA

In the same manner, we examine the revealed attention by adopting the most stringent criterion.

**Definition 7.** *For any $S \in \mathcal{X}$, $x$ is* ***revealed to catch attention in*** *$S$ if $x \in \Gamma_i(S)$ for all $i \in I$, and $x \in S$ is* ***revealed to catch no attention in*** *$S$ if $x \notin \Gamma_i(S)$ for all $i \in I$. Meanwhile, $S$ is* ***revealed to catch full attention*** *if $x$ is revealed to catch attention in $S$ for all $x \in S$, i.e., $\Gamma_i(S) = S$ for all $i \in I$.*

For the SIA, we can associate $S$ with one of its corresponding basic sets $B$, and interpret $B$ as the $\Gamma(S)$. As shown in Example 3, the DM's attention on $S$ is not necessarily basic. Intuitively, we can throw any "dominated" alternatives in any corresponding basic sets, and the new set is still consistent with one of the idempotent attention rules. Let us denote the undominated alternatives in $S$ as $U^c(S, P_R) = \{s \in S : \nexists\, x \in C(S)\ s.t.\ s\ P_R\ x\}$.

**Lemma 3.** *Suppose that $C$ admits an SIA. For any $T \subseteq S$, there is a $\Gamma_i$ such that $\Gamma_i(S) = T$ if and only if $C(T) = C(S)$ and $T \setminus C(S) \subseteq U^c(S, P_R)$.*

*Proof.* Suppose that $C$ admits an SIA. By the proof of Lemma 1, there is an $i \in I$ such that $\Gamma_i(\mathcal{X}) = \mathcal{B}$, and the corresponding consistent pair of the SIA is $\langle \Gamma_i, \succcurlyeq_i, \theta_i \rangle$. Take any $T \subseteq S$ with $C(T) = C(S)$ and $T \setminus C(S) \subseteq U^c(S, P_R)$. Without loss of generality, we can assume that $S, T \notin \mathcal{B}$ and $T \subset S$. For any $x \in X$, let us define $x^{\uparrow} =: \{y \in X : y\ P_R\ x\}$ and $x^{\downarrow} =: \{y \in X : x\ P_R\ y\}$. Moreover, let $x^{\uparrow c}$ and $x^{\downarrow c}$ as the complements of $x^{\uparrow}$ and $x^{\downarrow}$, repectively. Let us consider the collection $\bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$. Since $t^{\downarrow c}$ contains either $P_R$ dominating or incomparable alternatives with respect to $t$, we can find a linear completion of $P_R$ on this collection such that $t$ is worse than every $t' \in t^{\downarrow c}$. Let us denote it as $\overline{P_R}'$. Moreover, we can extend this binary relation by assuming that $x\ \overline{P_R}'\ y$ for $x \in X \setminus \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$ and $y \in \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$. Let us define a new binary relation

$$x \succcurlyeq_i' y := \begin{cases} x\ \overline{P_R}'\ y & if\ (x, y) \in \overline{P_R}', \\ x \succcurlyeq_i y & otherwise. \end{cases}$$

We then want to show that $\succcurlyeq_i'$ is a linear order. Obviously, $\succcurlyeq_i'$ is complete and antisymmetric. Take any $x, y, z \in Z$ with $x \succcurlyeq_i' y$ and $y \succcurlyeq_i' z$. It is clear that is $x, y, z \in \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$ or $x, y, z \in X \setminus \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$, $x \succcurlyeq_i' z$. When $x, y \in X \setminus \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$ and $z \in X \setminus \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$, $x \succcurlyeq_i z$. When $x \in X \setminus \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$ and $y, z \in X \setminus \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$, $x \succcurlyeq_i z$. Now, let us consider

$$\Gamma_i'(Y) = \begin{cases} T & if\ Y = S\ or\ T, \\ \Gamma_i(Y) & otherwise. \end{cases}$$

Obviously, $\Gamma_i'$ is idempotent. Let $\theta_i'(S) = min(C(S), \succcurlyeq_i')$. We want to show that $\langle \Gamma_i', \succcurlyeq_i', \theta_i' \rangle$ is a consistent pair of the SIA. Take any $(x, y) \in P_R$, we want to show that $(x, y) \in \succcurlyeq_i'$. If $x \in \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$, then $y \in \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$. As a result, $x\ \overline{P_R}'\ y$ which implies that $x \succcurlyeq_i' y$. If $x \in X \setminus \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$ and $y \in \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$, then $x \succcurlyeq_i' y$. If $x \in X \setminus \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$ and $y \in X \setminus \bigcup_{t \in T \setminus C(S)} t^{\downarrow c}$, then $x \succcurlyeq_i y$, which suggests that $x \succcurlyeq_i' y$. Consequently, for any

$Y \neq S$ *or* $T$, $C(Y) = \{s \in \Gamma_i(Y) : y \succcurlyeq_i \theta_i(Y)\} = \{s \in \Gamma'_i(Y) : y \succcurlyeq'_i \theta'_i(Y)\}$. We then only need to check the cases of $C(S)$ and $C(T)$. Since $T$ is not basic, we know that there is a $B \in \mathcal{B}(T)$ such that $\Gamma_i(T) = B$. Moreover, $C(T) = \{t \in B : t \succcurlyeq_i \theta_i(B)\} = C(B)$. Take any $t \in C(T)$, we know that $t \succcurlyeq'_i \theta'_i(T)$ by the definition of $\theta'_i$. Take any $t \in T$ with $t \succcurlyeq'_i \theta'_i(T)$, by contradiction, suppose that $t \notin C(T)$. As a result, $t \in T \setminus C(T) = T \setminus C(S)$. Therefore, for all $x \in C(S) = C(T) = C(B)$, we have either $x\ P_R\ t$ or $x$ and $t$ are $P_R$-incomparable. Hence, by the definition of $\succcurlyeq'_i$, we know that $x \succ'_i t$ which implies that $\theta'_i(T) \succ'_i t$.

For the converse direction, suppose that there is a $\Gamma_i$ such that $\Gamma_i(S) = T$. Thus, $C(S) = \{s \in T : s \succcurlyeq_i \theta_i(S)\} = C(T)$. By contradiction, suppose that $T \setminus C(S)$ is not a subset of $U^c(S, P_R)$. That is, there is a $t \in T \setminus C(S)$ such that there is a $x \in C(S)$ such that $t\ P_R\ x$ which implies that $t \succ_i x$. Therefore, $t \in C(S)$. □

The DM's attention for a menu $S$ can be assigned to one of its subsets $T$ with $C(S) = C(T)$. Moreover, $T$ is the union of a corresponding basic set of $S$ and a subset of $U^c(S, P_R)$. An alternative that attracts the attention of the DM in $S$ must exist in every corresponding basic set. In contrast, an alternative does not draw any attention in $S$, suggesting it is not in any $B \in \mathcal{B}(S)$ and dominates some of the chosen alternatives.

**Theorem 4.** *Suppose that $C$ is an SIA. An alternative $x$ is revealed to catch attention in $S$ if and only if $x \in B$ for every $B \in \mathcal{B}(S)$. An alternative $x \in S$ is revealed to catch no attention in $S$ if and only if for all $T \subseteq S$ that including $x$, either $C(T) \neq C(S)$ or $T \setminus C(S) \nsubseteq U^c(S, P_R)$.*

*Proof.* For the first half of the statements, suppose that $x$ is revealed to catch attention in $S$. That is, $x \in \Gamma_i(S)$ for all $i \in I$. As we know that for any $B \in \mathcal{B}(S)$ there is a $\Gamma_i$ such that $\Gamma_i(S) = B$, which implies that $x \in B$ for all $B \in \mathcal{B}(S)$. For the converse direction, suppose that $x \in B$ for every $B \in \mathcal{B}(S)$. By contradiction, supposes that there is an $i \in I$ such that $x \notin \Gamma_i(S)$. Since $C(S) = C(\Gamma_i(S))$, $\Gamma_i(S) \notin \mathcal{B}$. Take any $B' \in \mathcal{B}(\Gamma_i(S))$, we know that $B \subset \Gamma_i(S) \subseteq S$ and $C(B') = C(\Gamma_i(S)) = C(S)$. Hence, $B' \in \mathcal{B}(S)$ and $x \notin B'$. The second half of the statement is a direct result of Lemma 3. □

When analyzing revealed attention on menus, we seek to identify the characteristics of the menus that the DM pays full attention to. As shown in Proposition 3, basic sets must catch the DM's full attention. As well, the converse is true.

**Theorem 5.** *Suppose that $C$ is an SIA. $S$ is revealed to catch full attention if and only if $S$ is basic.*

*Proof.* We only need to show that if $S$ is revealed to catch full attention, then $S \in \mathcal{B}$. By contradiction, suppose that $S \in \mathcal{B}^c$. Then, there is a $B \in \mathcal{B}(S)$. Based on the proof of Lemma 1, there is an $i \in I$ such that $\Gamma_i(S) = B$. □

#### 3.2.3 Revealed Threshold under SIA

Threshold functions are the remaining part of revealing the SIA pair. We also employ the most stringent condition as usual.

**Definition 8.** *Suppose that $C$ can be rationalized by an SIA. An alternative $x \in S$ is **revealed to be the threshold of** $S$ if $\theta_i(S) = x$ for all $i \in I$.*

When the idempotent attention rule $\Gamma_i$ and the linear preference rule $\succcurlyeq_i$ are given, the least preferred chosen alternative serves as the threshold for every menu. The threshold function $\theta_i$ will then be uniquely pinned down under these circumstances.

**Proposition 5.** *If $C$ can be rationalized by an SIA under $\langle \Gamma_i, \succcurlyeq_i, \theta_i \rangle$, then $\theta_i(S)$ must be $\min(C(S), \succcurlyeq_i)$ for all $S \in \mathcal{X}$.*

*Proof.* Suppose that $C$ can be rationalized by an SIA under $\langle \Gamma_i, \succcurlyeq_i, \theta_i \rangle$. By contradiction, suppose that $\theta_i \neq \min(C(S), \succcurlyeq_i)$. If $\theta_i(S) \succ_i \min(C(S), \succcurlyeq_i)$, then $\min(C(S), \succcurlyeq_i) \notin C(S)$. If $\min(C(S), \succcurlyeq_i) \succ_i \theta_i(S)$, then $\theta_i(S) \in C(S)$. □

Due to the fact that every completion of $P_R$ can be regarded as a consistent linear preference, the threshold function also varies when the completion of $P_R$ changes. If an alternative $x \in C(S)$ dominates another chosen alternative, then it cannot be a candidate for $\theta_i(S)$ for every $i \in I$. Those candidates should be alternatives in $MIN(C(S), P_R) := \{x \in C(S) : \nexists y \in C(S) \ s.t. \ x \ P_R \ y\}$.

**Lemma 4.** *Suppose that $C$ can be rationalized by an SIA. For any $x \in X$ and $S \in \mathcal{X}$, $\theta_i(S) = x$ for some $i \in I$ if and only if $x \in MIN(C(S), P_R)$.*

*Proof.* Suppose that $C$ can be rationalized by an SIA, and take any $x \in S \in \mathcal{X}$. Assume that there is an $i \in I$ such that $\theta_i(S) = x$. By contradiction, suppose that $x \notin MIN(C(S), P_R)$, we then know that there is $y \in C(S)$ such that $x \ P_R \ y$. By Corollary 3, we know that $x \succcurlyeq_i y$ for all $i \in I$. Since $C(S) \subseteq \Gamma_i(S)$ for all $S$, $y \notin C(S)$. For the converse direction, suppose that $x \in MIN(C(S), P_R)$, we know that there is a linear order $\succcurlyeq_i$ which is a linear completion of $P_R$ such that $y \succcurlyeq_i x$ for all $y \in C(S)$. By the proof of

Theorem 1, we know that $\succsim_i$ is a consistent preference for the SIA. Based on Proposition 5, the corresponding $\theta_i(S) = x$. $\square$

As every alternative in $MIN(C(S), P_R)$ can be a candidate for the threshold used in $S$, it follows that the revealed threshold of $S$ is related to the items in $MIN(C(S), P_R)$.

**Theorem 6.** *Suppose that $C$ can be rationalized by an SIA. An alternative $x \in S$ is revealed to be the threshold of $S$ if and only if $y \ P_R \ x$ for every $y \in C(X)$ with $y \neq x$.*

# 4 On the Variations of Idempotent Attention Rules

It is important to note that attention rules are unique features of research on choices under limited attention. As far as we know, they all have some restrictions on attention formation. A popular attention rule is the attention filter, which was first used in Masatlioglu et al. (2012). Lleras et al. (2017) proposes an attention rule known as competition filters. Li (2023) combines attention filters with competition filters to form an attention rule called path independent consideration.

According to Li (2023), attention filters, competition filters, and path independent consideration are all special cases of idempotent attention rules. It should be noted, however, that not all attention rules in the previous research are idempotent. For example, the shortlisting procedure with capacity-$k$ (see Geng and Özbay (2021)) and weak competition filters (see Geng (2022)). The following section considers two special cases of the SIA: the satisficing under attention filter and the satisficing under competition filter.

## 4.1 Attention Filter

Attention filters mean that the DM is unable to distinguish any set between $S$ and $\Gamma(S)$. Formally,

**Definition 9.** *An attention rule $\Gamma$ is an **attention filter** if $\Gamma(T) = \Gamma(S)$ for every $S, T \in \mathcal{X}$ with $\Gamma(S) \subseteq T \subseteq S$.*

As with idempotent attention rules, attention filters can be interpreted similarly. When the DM is faced with a menu $T$ that lies between $S$ and $\Gamma(S)$. For the DM, $T$, $S$, and $\Gamma(S)$ are all identical. We will now examine satisficing under attention filters.

**Definition 10.** *A choice correspondence $C$ is a* ***satisficing under attention filter (SAF)*** *if $C$ is an SIA with $\langle \Gamma, \succsim, \theta \rangle$ where $\Gamma$ is an attention filter.*

The distinction between the SIA and the SAF comes from the differences between idempotent attention rules and attention filters. In idempotent attention rules, $\Gamma(S) = \Gamma(S \setminus (S \setminus \Gamma(S)))$ while attention filters require more. Specifically, $\Gamma(S) = \Gamma(S \setminus Y)$ for all $Y \subseteq S \setminus \Gamma(S)$. Therefore, when an alternative $x$ is removed from a menu $S$ and the DM's attention on $S \setminus x$ changes, $x$ must attract attention in $S$. If $x$ is not chosen by the DM, then all the alternatives in $S$ should be preferred over $x$. We can define this binary relation similarly to the SIA.

**Definition 11.** *For any $x, y \in X$, $x \ P^{AF} \ y$ if there is a set $S \in \mathcal{X}$ such that $x \in C(S)$, $y \notin C(S)$, and $C(S) \neq C(S \setminus y)$.*

Similar as the SIA, the acyclicity of $P^{CF}$ characterizes the SAF.

**Lemma 5.** *$C$ can be rationalized by an SAF if and only if $P^{AF}$ is acyclic.*

*Proof.* We first show the only if part. Assume that $C$ can be rationalized by an SAF. By contradiction, suppose that $P^{AF}$ is cyclic, i.e., there is a distinct pair of $x$ and $y$ such that $x \ P_R^{AF} \ y$ and $y \ P_R^{AF} \ x$ where $P_R^{AF}$ is the transitive closure of $P^{AF}$. We also know that $x$ must be revealed to be preferred to $y$, and $y$ is revealed to be preferred to $x$. Since $x$ and $y$ are distinct items, this observation contradicts the preference in any SAF is a linear order.

For the converse direction, assume that $P^{AF}$ is acyclic. Let $\succsim^{AF}$ be any completion of $P^{AF}$. Consider the following attention rule $\Gamma^{AF}$ and threshold function $\theta^{AF}$:

$$\theta^{AF}(S) = \min\left(C(S), \succsim^{AF}\right) \ , \textit{and} \ \Gamma^{AF}(S) = \{C(S)\} \cup \{s \in S : \theta^{AF}(S) \succsim^{AF} s\}.$$

Notice that $C(S) = \{x \in \Gamma^{AF}(S) : x \succsim^{AF} \theta(S)\}$ for all $S \in \mathcal{X}$. We then want to show that $C$ can be rationalized under the SIA by $\langle \Gamma^{AF}, \theta^{AF}, \succsim^{AF} \rangle$. We first show that $C(S \setminus y) = C(S)$ whenever $y \notin \Gamma^{AF}(S)$. By contradiction, suppose there is a $y \notin \Gamma^{AF}(S)$ such that $C(S) \neq C(S \setminus y)$. Take any $x \in C(S)$, we have $x \ P^{AF} \ y$ because $y \notin C(S)$ and $C(S) \neq C(S \setminus y)$. Consequently, we know that $\theta^{AF}(S) \succsim^{AF} y$ for $\succsim^{AF}$ is a completion of $P^{AF}$ which suggests that $y \in \Gamma^{AF}(S)$. Hence, we know that $C(S \setminus y) = C(S)$ whenever

$y \notin \Gamma^{AF}(S)$. Thus, we have $\theta^{AF}(S \setminus y) = \theta^{AF}(S)$ whenever $y \notin \Gamma^{AF}(S)$. Finally,

$$\begin{aligned}\Gamma^{AF}(S) = \Gamma^{AF}(S) &= \{C(S)\} \cup \{s \in S : \theta^{AF}(S) \succcurlyeq^{AF} s\} \\ &= \{C(S \setminus y)\} \cup \{s \in S : \theta^{AF}(S \setminus y) \succcurlyeq^{AF} s\} \\ &= \{C(S \setminus y)\} \cup \{s \in S \setminus y : \theta^{AF}(S \setminus y) \succcurlyeq^{AF} s\} \\ &= \Gamma^{AF}(S \setminus y).\end{aligned}$$

Since $\Gamma^{AF}$ is an attention filter, we can keep iteration to show that $\theta^{AF}(S) = \theta^{AF}(\Gamma^{AF}(S))$. As a result, $C$ can be rationalized by $\langle \Gamma^{AF}, \theta^{AF}, \succcurlyeq^{AF} \rangle$ under the SAF. □

Similarly to the SIA, it is also possible to form a WARP-like axiom analogously to that in Masatlioglu et al. (2012).

**Axiom 2** (WARP-AF)**.** *For every $S \in \mathcal{X}$ there is an $x^* \in S$ such that for every $T$ if $C(T) \cap S \neq \emptyset$ and $C(T \setminus x^*) \neq C(T)$, then $x^* \in C(T)$.*

$x^*$ can still be interpreted as the "best" alternatives in $S$. Due to the fact that $C$ is a choice correspondence in our framework, our WARP-like axiom modifies WARP(LA) in Masatlioglu et al. (2012) by allowing $C(T) \cap S \neq \emptyset$. As it turns out, the acyclicity of $P^{CF}$ is equivalent to the WARP-AF.

**Lemma 6.** *$P^{AF}$ is acyclic if and only if $C$ satisfies WARP-AF.*

*Proof.* If $P^{AF}$ is acyclic, then $C$ can be rationalized by a consistent SLA pair $\langle \Gamma, \succcurlyeq, \theta \rangle$. Let $x^* = \max(S, \succcurlyeq)$ for all $S \in \mathcal{X}$. Take any $T$ where $C(T) \cap S \neq \emptyset$ and $C(T \setminus x^*)$. If $x^* \in C(T)$, we are done. If there is a $y \in C(T) \cap S$, we have $x^* \succcurlyeq y$. Because$C(T \setminus x^*) \neq C(T)$ implies that $x^* \in \Gamma(T)$, we have $x^* \in C(T)$.

For the converse direction, we prove it by showing its contrapositive statement. Suppose that $P^{AF}$ is cyclic, we then know that there is a $z_1 \in X$ such that $z_1 \; P_R^{AF} \; z_1$, i.e., there are finite collections $\{z_j\}_{j=1}^m$ $\{Z_j\}_{j=1}^m$ where $z_j \in Z_j$ for each $j$ such that

$$\begin{aligned}&z_1 \in C(Z_1), C(Z_1) \neq C(Z_1 \setminus z_2) \textit{ and } z_2 \notin C(Z_1), \\ &z_2 \in C(Z_2), C(Z_2) \neq C(Z_2 \setminus z_3) \textit{ and } z_3 \notin C(Z_2), \\ &\dots \\ &z_m \in C(Z_m), C(Z_m) \neq C(Z_m \setminus z_1) \textit{ and } z_1 \notin C(Z_m).\end{aligned}$$

Let $Z = \{z_j\}_{j=1}^m$. For any $z_1 \in Z$ there is a set $Z_m$ such that $C(Z_m) \cap S \neq \emptyset$, $C(Z_m \setminus z_1) \neq C(Z_m)$, and $z_1 \notin C(Z_m)$. For any $z_j \in Z$ with $j \geq 2$, there is a set $Z_{j-1}$ such that $C(Z_{j-1}) \cap S \neq \emptyset$, $C(Z_{j-1} \setminus z_j) \neq C(Z_{j-1})$. □

Consequently, the SAF can be characterized by the WARP-AF as well.

**Theorem 7.** *A choice correspondence $C$ is an SAF if and only if $C$ satisfies WARP-AF.*

### 4.2 Competition Filter

Lleras et al. (2017) defines competition filters, which suggest that if an alternative catches the DM's attention in a menu $S$, it will also do so in all subsets of $S$ that include $x$.

**Definition 12.** *An attention rule is a competition filter if $x \in \Gamma(S) \cap T$ implies that $x \in \Gamma(T)$ for all $T, S \in \mathcal{X}$ with $T \subseteq S$.*

In a similar manner to the SAF, satisficing under competition filter can be defined.

**Definition 13.** *A choice correspondence $C$ is a **satisficing under competition filter (SCF)** if $C$ is an SIA with $\langle \Gamma, \succcurlyeq, \theta \rangle$ where $\Gamma$ is a competition filter.*

Competition filters indicate that, if the DM pays attention to some alternatives in a larger menu, these alternatives, if available in a smaller menu, should attract the DM's attention. This is because perceiving alternatives in a more complex environment should be more difficult. The challenge is to determine how to infer the DM's attention from a menu. One trivial observation is that the chosen alternatives must catch the DM's attention. Thus, we can define a binary relation based on it.

**Definition 14.** *For any $x, y \in X$, $x\ P^{CF}\ y$ if there are sets $T, S \in \mathcal{X}$ with $\{x, y\} \subseteq T \subseteq S$ such that $y \in C(S)$, $x \in C(T)$ and $y \notin C(T)$.*

The acyclicity of $P^{CF}$ characterizes the SCF.

**Lemma 7.** *A choice correspondence $C$ is an SCF if and only if $P^{CF}$ is acyclic.*

*Proof.* Assume that $C$ can be rationalized by an SCF pair $\langle \Gamma, \theta, \succcurlyeq \rangle$. By contradiction, suppose that there is a $z_1 \in X$ such that $z_1\ P_R^{CF}\ z_1$ where $P_R^{CF}$ is the transitive closure of $P^{CF}$. Take any $x\ P^{CF}\ y$, we know that $x \succcurlyeq y$ by the definition of $P^{CF}$. Since $\succcurlyeq$ is a linear order, we know that $z_1\ P_R^{CF}\ z_1$ cannot be true.

Suppose that $P^{CF}$ is acyclic for the converse direction. We then can fix any linear order $\succsim^{CF}$ who is a completion of $P_R^{CF}$. Consider

$$\Gamma^{CF}(T) = \{x \in T : x \in C(S) \ for\ some\ S \supseteq T\} \ and\ \theta^{CF}(T) = min(C(T), \succsim^{CF}).$$

It is clear that $\Gamma^{CF}$ is a competition filter. We then want to show that $\langle \Gamma^{CF}, \succsim^{CF}, \theta^{CF} \rangle$ is a consistent SAF pair, i.e., $C(T) = \left\{t \in \Gamma^{CF}(T) : t \succsim^{CF} \theta^{CF}(T)\right\}$. Take any $t \in C(T)$, we know that $t \in \Gamma^{CF}(T)$ and $t \succsim^{CF} \theta^{CF}(T)$ by the definitions. For the converse direction, take any $t \in \Gamma^{CF}(T)$ and $t \succsim^{CF} \theta^{CF}(T)$. By contradiction, suppose that $t \notin C(T)$. We know that there is a set $S \supset T$ such that $x \in C(S)$, which implies that $\theta^{CF}(T)\ P^{CF}\ x$. Hence $\theta^{CF}(T) \succsim^{CF} x$. □

The WARP-like axiom for the SCF also can be formed as in the SIA and the SAF.

**Axiom 3.** *(WARP-CF) For every $S \in \mathcal{X}$, there is a $x^* \in S$ such that for any $T$ with $x^* \in T$, if $C(T) \cap S \neq \emptyset$ and $x^* \in C(Y)$ for some $Y \supseteq T$ then $x^* \in C(T)$.*

Similarly, $x^*$ can be interpreted as the "best" alternatives in $S$. When an alternative in $S$ is chosen in $T$, and $x^*$ catches the DM's attention in $T$, the $x^*$ should be chosen in $T$. The acyclicity of $P^CF$ is equivalent to WARP-CF.

**Lemma 8.** *$P^{CF}$ is acyclic if and only if $C$ satisfies WARP-CF.*

*Proof.* We first suppose that $P^{CF}$ is acyclic. We then know that $C$ is an SCF under a consistent pair $\langle \Gamma, \theta, \succsim \rangle$. Take any $S \in \mathcal{X}$, and $x^* = max(S, \succsim)$. For any $T \in \mathcal{X}$, since there is a $Y \supseteq T$ such that $x^* \in C(Y)$, $x^* \in \Gamma(T)$. Because $C(T) \cap S \neq \emptyset$, we know that $x^* \in C(T)$.

For the converse direction, we show it by showing the contrapositive statement. Suppose that $P^{CF}$ is acyclic. There is a finite sequence of $\{z_j\}_{j=1}^m$ with $z_1\ P^{CF}\ z_2, ..., \ P^{CF}\ z_m\ P^{CF}\ z_1$. That is there are finite collections of menus $\{Z_j\}_{j=1}^m$ and $\{Z'_j\}_{j=1}^m$ such that

$$\begin{aligned}
&\{z_1, z_2\} \subseteq Z_1 \subseteq Z'_1,\ z_2 \in C(Z'_1),\ z_1 \in C(Z_1),\ and\ z_2 \notin C(Z_1),\\
&\{z_2, z_3\} \subseteq Z_2 \subseteq Z'_2,\ z_3 \in C(Z'_2),\ z_2 \in C(Z_2),\ and\ z_3 \notin C(Z_2),\\
&\dots\\
&\{z_m, z_1\} \subseteq Z_m \subseteq Z'_m,\ z_1 \in C(Z'_m),\ z_m \in C(Z_m),\ and\ z_1 \notin C(Z_m).
\end{aligned}$$

Let's consider $Z = \{z_j\}_{j=1}^m$. For $z_1$, there is a $Z_m \subseteq Z'_m$ such that $z_1 \in C(Z'_m)$, $C(Z_m) \cap Z \neq \emptyset$ and $z_1 \notin C(Z_m)$. For any $z_j$ with $j > 1$, there is a $Z_{j-1} \subseteq Z'_{j-1}$ such that $z_j \in C(Z'_{j-1})$, $C(Z_{j-1}) \cap Z \neq \emptyset$ and $z_j \notin C(Z_{j-1})$. □

Obviously, we can characterize the SCF by WARP-CF.

**Theorem 8.** *A choice correspondence $C$ is an SCF if and only if $C$ satisfies WARP-CF.*

## 5 Conclusion

We propose a satisficing model under idempotent attention in this paper. To characterize the SIA, we take perspectives from both the acyclicity condition of some binary relation and the WARP-like axiom.

SIA separates limited attention from limited cognitive abilities, which have long been considered as factors leading to satisficing or bounded rationality (see Simon (1997), Tyson (2008), Frick (2016) et.al.). In particular, we consider a family of idempotent attention rules used in Li (2023), which is a generalization of attention filters (see Masatlioglu et al. (2012)), competition filters (Lleras et al. (2017)), and path independent consideration (see Lleras et al. (2021)). Two systems of characterization have been provided for both the SAF and SCF as special cases.

We analyze the revealed SIA pairs under the strictest conditions to determine the welfare implications of the SIA. Unfortunately, if a choice correspondence is an SIA, then it must have at least two consistent SIA pairs. This negative result is due to the inability to determine the DM's preference based on the choice correspondence.

# Appendix

## A Characterization of Satisficing under Full Attention

We first introduce the formal definition of satisficing (under full attention).

**Definition 15.** *A choice correspondence $C$ is a* ***satisficing (under full attention)*** *if there is a linear order $\succcurlyeq$ and a threshold function $\theta : \mathcal{X} \to X$ such that $C(S) = \{s \in S : s \succcurlyeq \theta(S)\}$ where $\theta(S) \in S$ for all $S \in \mathcal{X}$.*

According to the satisfaction model, the chosen alternative should be better than the unchosen alternative. We can define it as a binary relation.

**Definition 16.** *For any $x, y \in X$, $x \ P^s \ y$ if there is an $S \in \mathcal{X}$ such that $x \in C(S)$ and $y \in S \setminus C(S)$.*

As shown in Aleskerov et al. (2007) and Tyson (2008), satisficing can be described as the following Lemma. We still show the proof in detail despite the slight difference in our settings.

**Lemma 9.** *A choice correspondence can be rationalized by a satisficing model if and only if $P^s$ is acyclic.*

*Proof.* Take an arbitrary choice correspondence $C$, and suppose that it can be rationalized by a satisficing model under $\langle \succcurlyeq, \theta \rangle$. By contradiction, suppose that there is a finite collection of alternatives $\{x_i\}_{i=1}^k$ such that $x_1 \ P^s \ x_2 \ P^s, ..., P^s \ x_k \ P^s x_1$. There is a collection of sets $\{S_i\}_{i=1}^k$ such that $x_i \in C(S_i)$ and $x_{i+1} \in S_i \setminus C(S_i)$ for all $i < k$, and $x_k \in C(S_k)$ and $x_1 \in B_k \setminus C(B_k)$. As a result, $\succcurlyeq$ is not linear.
For the converse direction, suppose that $P^s$ is acyclic. Let $P^s_R$ be the transitive closure of $P^s$, $\overline{P^s_R}$ be a completion of $P^s_R$. Let $f(S) = min\{C(S), \overline{P^s_R}\}$. It is evident that $x \in C(S)$ implies that $x \ \overline{P^s_R} \ f(S)$. Suppose that there is a $t \in S$ with $t \ \overline{P^s_R} \ f(S)$ such that $t \notin C(S)$. We then know that $f(S) \ P^s \ t$. Consequently, $C(S) = \big\{s \in S : s \ \overline{P^s_R} \ f(S)\big\}$. □

We remain to show that WARP-S is equivalent to the acyclicity of $P^s$.

**Lemma 10.** *$P^s$ is acyclic if and only if $C$ satisfies WARP-S.*

*Proof.* Suppose that $P^s$ is acyclic. By Lemma 9, $C$ is admits a satisficing under $\langle \succcurlyeq, \theta \rangle$. Take any $S \in \mathcal{X}$, let $x^* = max(S, \succcurlyeq)$. Fix any $T \in \mathcal{X}$ with $x^* \in T$, if $t \in C(T) \cap S$, then

$t \succcurlyeq \theta(T)$. Also, $t \in S$ suggests that $x^* \succcurlyeq t \succcurlyeq \theta(S)$. As a result, $x^* \in C(T)$.
For the converse direction, suppose that $P$ is cyclical, i.e., there is a collection of $\{S_i\}_{i=1}^k$ such that

$$
\begin{aligned}
&s_1, s_2 \in S_1, s_1 \in C(S_1),\ \textit{and}\ s_2 \notin C(S_1);\\
&s_2, s_3 \in S_2, s_2 \in C(S_2),\ \textit{and}\ s_3 \notin C(S_2);\\
&\dots\\
&s_1, s_k \in S_k, s_k \in C(S_k),\ \textit{and}\ s_1 \notin C(S_k).
\end{aligned}
$$

Let $S = \{s_i\}_{i=1}^k$. We cannot find a corresponding $x^*$ in WARP-S for $S$. □

The following theorem is a direct result of Lemma 9 and Lemma 10.

**Theorem 9.** *A choice correspondence $C$ admits satisficing if and only if it satisfies WARP-S.*

## References


Fuad Aleskerov, Denis Bouyssou, and Bernard Monjardet. *Utility maximization, choice and preference*. Springer, Berlin, second edition, 2007.

B Douglas Bernheim and Antonio Rangel. Toward choice-theoretic foundations for behavioral welfare economics. *American Economic Review*, 97(2):464–470, 2007.

B Douglas Bernheim and Antonio Rangel. Beyond revealed preference: choice-theoretic foundations for behavioral welfare economics. *The Quarterly Journal of Economics*, 124 (1):51–104, 2009.

Peter C. Fishburn. Semiorders and choice functions. *Econometrica*, 43(5-6):975–977, 1975.

Mira Frick. Monotone threshold representations. *Theoretical Economics*, 11(3):757–772, 2016.

Sen Geng. Limited consideration model with a trigger or a capacity. *Journal of Mathematical Economics*, 101:102692, 2022.

Sen Geng and Erkut Y Özbay. Shortlisting procedure with a limited capacity. *Journal of Mathematical Economics*, 94:102447, 2021.

Dayang Li. Additive representation under idempotent attention. Working paper, Department of Economics, University of California Riverside, 2023.

Juan Lleras, Yusufcan Masatlioglu, Daisuke Nakajima, and Erkut Ozbay. Path-independent consideration. *Games*, 12(1):21, 2021.

Juan Sebastian Lleras, Yusufcan Masatlioglu, Daisuke Nakajima, and Erkut Y Ozbay. When more is less: Limited consideration. *Journal of Economic Theory*, 170:70–85, 2017.

R. Duncan Luce. Semiorders and a theory of utility discrimination. *Economitrica*, 24(2): 178–191, 1956.

Paola Manzini and Marco Mariotti. Choice by lexicographic semiorders. *Theoretical Economics*, 7(1):1–23, 2012.

Yusufcan Masatlioglu, Daisuke Nakajima, and Erkut Y Ozbay. Revealed attention. *American Economic Review*, 102(5):2183–2205, 2012.

Herbert A. Simon. Theories of decision-making in economics and behavioral science. *American Economic Review*, 49(3):253–283, 1959.

Herbert A. Simon. *Models of bounded rationality. 2. Behavioral economics and business organization.* MIT Press, 1982.

Herbert A. Simon. *Models of bounded rationality: Empirically grounded economic reason*, volume 3. MIT press, 1997.

Christopher J Tyson. Cognitive constraints, contraction consistency, and the satisficing criterion. *Journal of Economic Theory*, 138(1):51–70, 2008.